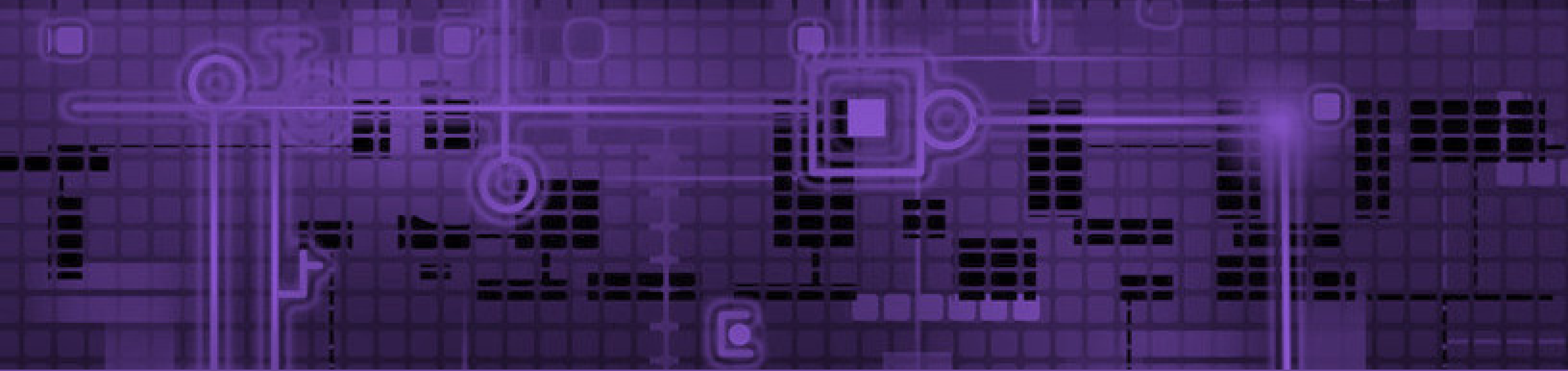

Optimization of Evaporator to Condenser

Length Ratio for Heat Pipe Maximum Performance

Introduction

Heat pipes have a multitude of applications in the thermal management of electronics. The thermal resistance of a heat pipe strongly depends, among other parameters, on so-called L-ratio, which is the ratio between the evaporator length and condenser length. The optimal L-ratio is defined as being the value for which the heat pipe thermal resistance is minimal, for a certain heat pipe assembly configuration. Finding the optimal L-ratio offers a valuable tool for thermal practitioners to most effectively deploy the heat pipe in their application.

This article reviews an analytical approach for finding this optimum L-ratio [1]. The formula obtained is applied to round heat pipes frequently used in electronics cooling.

Nomenclature

a_f = annular fin surface area, m^2

A = constant

A_f = is heat transfer area at condenser section, finned heat pipe, m^2

A_p = is heat transfer area at condenser section, bare heat pipe, m^2

C_1, C_2, C_3, C_4, C_5, C_6, C_7 = constants

C_A, C_B = compounded constants

d_i = heat pipe inner diameter, m

d_o = heat pipe outer diameter, m

d_{iw} = wick structure inner diameter, m

d_{ow} = wick structure outer diameter, m

d_{me} = mean diameter of bonding material at evaporator section, m

d_{mc} = mean diameter of bonding material at condenser section, m

D = annular fin diameter or equivalent diameter of a rectangular fin, m^2

h = equivalent heat transfer coefficient of bare pipe to finned heat pipe at condenser section, $W/m^2 \bullet {}^oC$

h_f = heat transfer coefficient of finned heat pipe at condenser section, $W/m^2 \bullet {}^oC$

k_{eff} = effective thermal conductivity of liquid saturated wick structure, $W/m \bullet {}^oC$

k_p = thermal conductivity of pipe wall, $W/m \bullet {}^oC$

K_b = thermal conductivity of heater block material, $W/m \bullet {}^oC$

K_{bm} = thermal conductivity of bonding material, $W/m \bullet {}^oC$

L = total heat pipe length, m

L_a = length of adiabatic section, m

L_c = length of condenser section, m

L_e = length of evaporator section, m

L_h = length of heat source, m

P = fin pitch, m

R = total thermal resistance of the heat pipe, °C/W

R_{sp} = spreading thermal resistance at the evaporator section, °C/W

R_{eb} = heater block thermal resistance, °C/W

R_{ebm} = thermal resistance of bonding material at evaporation section, °C/W

R_{ep} = thermal resistance of radial conduction of pipe wall at evaporation section, °C/W

R_{ew} = thermal resistance of radial conduction of liquid/wick combination at evaporation section, °C/W

R_{ee} = vaporization thermal resistance, °C/W

R_{cc} = condensation thermal resistance, °C/W

R_{cw} = thermal resistance of radial conduction of liquid/wick combination at condenser section, °C/W

R_{cp} = thermal resistance of radial conduction of pipe wall at condenser section, °C/W

R_{ebm} = thermal resistance of bonding material at condenser section, °C/W

R_{sk} = thermal resistance of forced air convection at heat sink to ambient, °C/W

R_v = vapor thermal resistance, °C/W

R_{aw} = thermal resistance of axial conduction of liquid/wick combination, °C/W

R_{ap} = thermal resistance of axial conduction of pipe wall, °C/W

S = length or width of an equivalent square heat source, m

S_f = fin to fin gap, m

t_b = thickness of the heater block at the evaporator section, m

t_f = fin thickness, m

t_{bm} = thickness of bonding material, m

W_b = width of the heater block, m

W_h = width of the heat source, m

Heat Pipe Thermal Resistance

A typical cylindrical heat pipe assembly is shown in Figure 1 [1]. The assembly consists of a heat pipe with heating and cooling sections attached to it. The length of the evaporator (L_e) and the length of the condenser (L_c) can be varied.

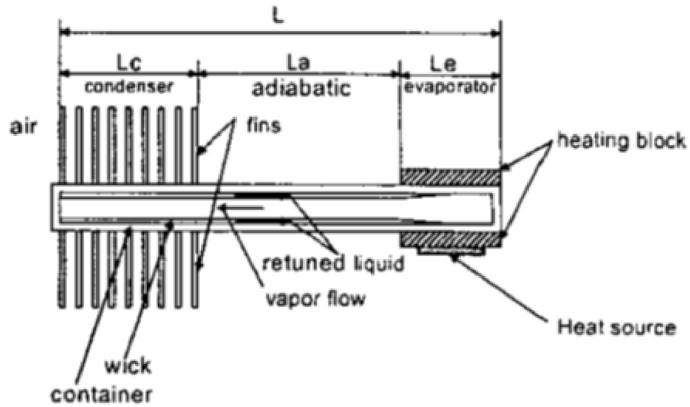

Figure 1. Schematic of a Heat Pipe Assembly [1]

The thermal resistance network for the above assembly is presented in Figure 2.

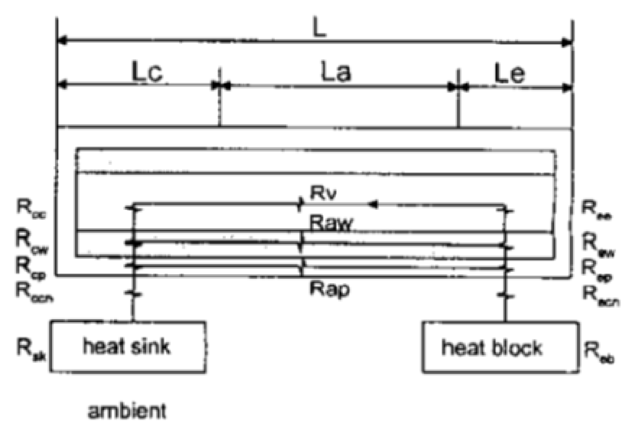

Figure 2. Thermal Resistance Network for a Heat Pipe Assembly [1]

Table 1 includes order of magnitude estimations of several thermal resistances from the network in Figure 2.

Thermal Resistance	°C/W
R_{ep} and R_{cp}	10^{-1}
R_{ew} and R_{cw}	10^{+1}
R_{ee} and R_{cc}	10^{-5}
R_v	10^{-8}
R_{ap}	10^{+2}
R_{aw}	10^{+4}

Table 1. Magnitude Comparison of Thermal Resistances in Heat Pipes [2]

By analyzing the values in Table 1, the circuits including R_{ap} and R_{aw} can be eliminated, since both these resistances are significantly larger than the others. Also, since R_{ee}, R_{cc} and R_v are relatively small, they can be neglected. Therefore, the total thermal resistance of the heat pipe (R) can be expressed as:

$$R = R_{sp} + R_{eb} + R_{ebm} + R_{ep} + R_{ew} + R_{cw} + R_{cp} + R_{cbm} + R_{sk} \tag{1}$$

The factors on the right hand side can be expressed as:

$$R_{sp} = C_1 Ln(L_e) + A \tag{2}$$

$$R_{eb} = \frac{t_b}{L_e \cdot W_b \cdot K_b} \tag{3}$$

$$R_{ebm} = \frac{t_{bm}}{L_e \cdot \pi \cdot d_{me} \cdot K_{bm}} \tag{4}$$

$$R_{ep} = \frac{1}{2 \cdot \pi \cdot k_p \cdot L_e} 1n\left(\frac{d_o}{d_i}\right) \tag{5}$$

$$R_{ew} = \frac{1}{2 \cdot \pi \cdot k_{eff} \cdot L_e} 1n\left(\frac{d_{ow}}{d_{iw}}\right) \tag{6}$$

$$R_{cw} = \frac{1}{2 \cdot \pi \cdot k_{eff} \cdot L_c} 1n\left(\frac{d_{ow}}{d_{iw}}\right) \tag{7}$$

$$R_{cp} = \frac{1}{2 \cdot \pi \cdot k_p \cdot L_c} 1n\left(\frac{d_o}{d_i}\right) \tag{8}$$

$$R_{cbm} = \frac{t_{bm}}{L_c \cdot \pi \cdot d_{mc} \cdot K_{bm}} \tag{9}$$

$$R_{sk} = \frac{1}{A_p \cdot h} = \frac{1}{L_c \cdot \pi \cdot d_o \cdot h} \tag{10}$$

For a given heater block and heat source size, the spreading resistance R_{sp} will be only a function of L_e, hence equation (2). The equivalent heat transfer coefficient at the condenser section can be

$$h = \frac{A_f \cdot h_f}{A_p} \tag{11}$$

expressed as:
Assuming the fins at the condenser section are annular, the total area for a fin (neglecting fin tip

$$a_f = \frac{\pi}{2} \cdot (D^2 - d_o^2) \tag{12}$$

area) can be calculated as:
The total fin and pipe areas are, respectively:

$$A_f = \frac{L_c}{p}\left[\frac{\pi}{2} \cdot (D^2 - d_o^2) + \pi \cdot d_o \cdot S_f\right] \tag{13}$$

$$A_p = \pi \cdot d_o \cdot L_c \tag{14}$$

Some geometrical parameters mentioned above are illustrated in Figure 3.

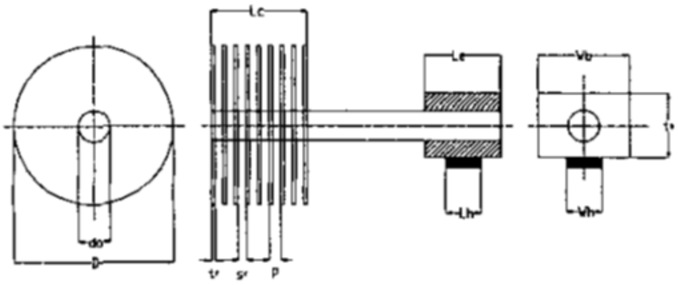

Figure 3. Geometrical Parameters for Heat Pipe Assembly Analysis [1]

Using (13) and (14), equation (11) can be

$$h = \frac{\frac{\pi \cdot L_c}{p}\left[\frac{1}{2}\left(D^2 - d_o^2\right) + D \cdot t_f \cdot d_o \cdot S_f\right] \cdot h_f}{\pi \cdot d_o \cdot L_c} \quad (15)$$

$$= \frac{\left[0.5\left(D^2 - d_o^2\right) + D \cdot t_f \cdot d_o \cdot S_f\right] h_f}{d_o \cdot p}$$

rearranged as:
Therefore, the overall thermal resistance can be written as:

$$R = [C_1 \cdot 1n\,(L_e) + A] + \frac{C_2}{L_e} + \frac{C_3}{L_e} + \frac{C_4}{L_e} + \frac{C_5}{L_e} + \frac{C_6}{L_e} + \frac{C_6}{L_c} + \frac{C_5}{L_c} + \frac{C_4}{L_c} + \frac{C_3}{L_c} + \frac{C_7}{L_c} \quad (16)$$

where C_2, C_3, C_4, C_5, C_6, C_7 are constants for a particular heat pipe assembly [1].

So, for a specific heat pipe included in a specific assembly we can write:

$$R = F(L_e, L_c) \quad (17)$$

Since

$$L_c = L - (L_a + L_e) \quad (18)$$

equation (18) can be expressed as well as:

$$R = F\,[L_e, L(L_e)] \quad (19)$$

Optimal L-ratio

Optimal L-ratio (L_e/L_c) is attained when heat pipe thermal resistance is minimal. Therefore, by setting:

$$\frac{dR}{dL_e} = 0 \quad (20)$$

the minimum of the function will be found. Figure 4 illustrates equation (17) between two set values

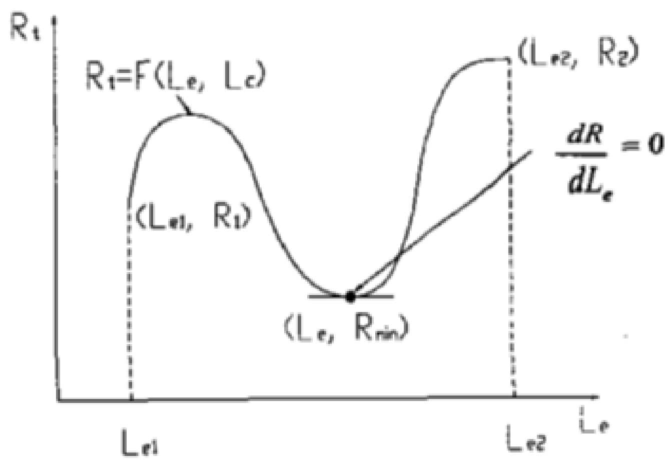

Figure 4. Minimum Thermal Resistance as a Function of Evaporator Length [1]

for evaporator length.

After appropriate manipulations equation (20) yields:

$$\frac{C_A}{L_c^2} = \frac{C_b - C_1 \cdot L_e}{L_e^2} \quad (21)$$

or

$$L_e = [L_c\,(C_1^2 L_c^2 + 4C_A C_B)^{0.5} - L_c^2 C_1]/(2C_A) \quad (22)$$

where:

$$C_1 = 0.0179W^{-0.1244}\ 2.564e^{-132.92t_b}\ 0.0863S^{-0.535}\ K_b/390 \quad _b \tag{23}$$

$$S = (L_h W_h)^{0.5} \tag{24}$$

$$C_A = C_3 + C_4 + C_5 + C_6 + C_7 \tag{25}$$

$$C_b = C_2 + C_3 + C_4 + C_5 + C_6 \tag{26}$$

The analytical expression obtained above was applied for round heat pipes with outer diameters of 4, 5 and 6 mm and different condenser lengths. Table 2 includes results for a 4 mm OD heat pipe, 10 x 10 mm heat source and a heater block 30 x 30 x 6 mm. At the condenser end, fins were 0.3 mm thick, fin gap 1 mm and fin diameter 40 mm. The optimal L_e/L_C varies between 0.4 and 0.6.

L_c(m)	L_e(m)	C_1	C_A	C_B	C_A/L_c	C_B-C_1*L_e)/L_e	L_e/L_c
0.200	0.075	0.032	0.012	0.004	0.30	0.3	0.4
0.100	0.046	0.032	0.012	0.004	1.21	1.21	0.5
0.060	0.030	0.032	0.012	0.004	3.37	3.37	0.5
0.040	0.021	0.032	0.012	0.004	7.58	7.58	0.5
0.020	0.011	0.032	0.012	0.004	30.34	30.33	0.6

Table 2. Optimal L_e/L_c for 4 mm OD Heat Pipe Assembly [1]

Table 3 includes results for a 5 mm OD heat pipe, 10 x 10 mm heat source and a heater block 35 x 35 x 7 mm. Fins were 0.3 mm thick, fin gap 1.2 mm and fin diameter 40 mm. Similarly, the optimal varies between 0.4 and 0.5.

L_c(m)	L_e(m)	C_1	C_A	C_B	C_A/L_c^2	C_B-C_1*L_e)/L_e^2	L_e/L_c
0.200	0.071	0.028	0.011	0.003	0.27	0.27	0.4
0.100	0.044	0.028	0.011	0.003	1.06	1.06	0.4
0.060	0.029	0.028	0.011	0.003	2.95	2.95	0.5
0.040	0.020	0.028	0.011	0.003	6.64	6.64	0.5
0.020	0.011	0.028	0.011	0.003	26.54	26.54	0.5

Table 3. Optimal L_e/L_c for 5 mm OD Heat Pipe Assembly [1]

The results for a 6 mm OD heat pipe are presented in Table 4. The heat source was 10 x 10 mm and

L_c(m)	L_e(m)	C_1	C_A	C_B	C_A/L_c^2	C_B-C_1*L_e)/L_e^2	L_e/L_c
0.200	0.068	0.025	0.012	0.003	0.29	0.29	0.3
0.100	0.041	0.025	0.012	0.003	1.15	1.17	0.4
0.060	0.027	0.025	0.012	0.003	3.21	3.21	0.4
0.040	0.019	0.025	0.012	0.003	7.22	7.22	0.5
0.020	0.010	0.025	0.012	0.003	28.87	28.87	0.5

Table 4. Optimal L_e/L_c for 6 mm OD Heat Pipe Assembly [1]

the heater block 40 x 40 x 8 mm. Fins were 0.4 mm thick, fin gap 1.8 mm and fin diameter 40 mm. In this case the optimal varies between 0.3 and 0.5.

Conclusions

An analytical formula for calculating the optimal evaporator to condenser ratio (L_e/L_c) has been deduced. The formula is a valuable tool for thermal engineers, offering a simple rule for choosing appropriate L_e and L_c values.

The formula was applied for several frequently used cylindrical heat pipes, yielding optimal L-ratios between 0.3 and 0.6. However, these results are valid for a fixed value of $L_e + L_c$. For maximum heat transfer rate (minimum thermal resistance), $L_e + L_c$ should equal the heat pipe length, which means no adiabatic section. Also, the formula needs to be validated through experimental results and generalized (for different heat pipe cross sections and heat pipe assembly orientations).

References:

1. Wu, X.P., Mochizuki, M., Saito, Y., Nguyen, T., Wuttijumnong, V., and Wu, D.,"Analyzing and Modeling on Optimized L-Ratio of Evaporator Section to Condenser Section for Micro Heat Pipe Heat Sinks", 19th IEEE SEMI-THERM Symposium, 2003.

2. Peterson, G.P.,"An Introduction to Heat Pipes, Modeling, Testing and Applications", John Wiley & Sons, Inc., pp.147-150, 1994.